Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

: explanation

∵ is 'because'

∴ is 'therefore'

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA11/01**

Mathematics

June 2022

International Advanced Subsidiary/Advanced Level

Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

P69458A

Q:1/1/1/1/

P69458A0132

Pearson

1. Find

$$\int\left(10x^5 + 6x^3 - \frac{3}{x^2}\right)dx$$

giving your answer in simplest form.

(4)

Integration Method

① Write in easier form for integration

$10x^5 + 6x^3 - \frac{3}{x^2} = 10x^5 + 6x^3 - 3x^{-2}$

↳ indicies rule : $\frac{a}{x^b} = ax^{-b}$

② Integration

$$\int 10x^5 + 6x^3 - 3x^{-2}\,dx = \left[\left(\frac{10}{5+1}x^{5+1}\right) + \left(\frac{6}{3+1}x^{3+1}\right) + \left(\frac{-3}{-2+1}x^{-2+1}\right)\right]$$

$= \frac{10}{6}x^6 + \frac{6}{4}x^4 + \frac{-3}{-1}x^{-1} + C$

↳ DON'T FORGET !!! or will lose a mark.

$= \frac{5}{3}x^6 + \frac{3}{2}x^4 + 3x^{-1} + C$

$\therefore \frac{5}{3}x^6 + \frac{3}{2}x^4 + 3x^{-1} + C$

↳ also written as $\frac{3}{x}$

P 6 9 4 5 8 A 0 2 3 2

Leave blank

Question 1 continued

Q1

(Total 4 marks)

2. In the triangle ABC,

- $AB = 21$ cm
- $BC = 13$ cm
- angle $BAC = 25°$
- angle $ACB = x°$

unit is degrees

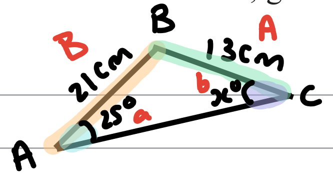
(a) Use the sine rule to find the value of $\sin x°$, giving your answer to 4 decimal places. **(2)**

asking for $\sin x°$ NOT $x°$

Given also that AB is the longest side of the triangle,

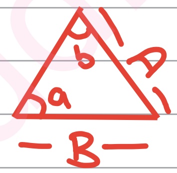
(b) find the value of x, giving your answer to 2 decimal places. **(3)**

a)



Sine rule :

$$\frac{\sin a}{A} = \frac{\sin b}{B}$$



$$\frac{\sin 25°}{13} = \frac{\sin x°}{21} \Rightarrow 21\left(\frac{\sin 25}{13}\right) = \sin x°$$

$$\sin x° = \frac{21 \sin 25°}{13} = 0.682691\ldots \approx 0.6827°$$

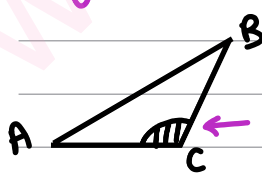
$\therefore \sin x° = 0.6827°$ (4dp)

b) $\sin x° = 0.6827$ ← from part (a)

inverse sine $x° = \sin^{-1}(0.6827)$ inverse sine

$$x = 43.0549925\ldots \approx 43.055$$

AB is longest side of triangle, so AC < AB. This causes angle <ACB to be LARGER than <ABC.



(angle > 90°)

<ACB is OBTUSE

Sine can give 2 answers.
∴ we can subtract acute angle given from 180°.

$\therefore x = 180 - 43.055 = 136.945 \approx 136.95$

$\therefore x = 136.95°$ (2dp)

Leave blank

Question 2 continued

Q2

(Total 5 marks)

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(i) Show that $\dfrac{\sqrt{180}-\sqrt{80}}{\sqrt{5}}$ is an integer and find its value.

↳ whole number

(2)

(ii) Simplify

$$\frac{4\sqrt{5}-5}{7-3\sqrt{5}}$$

giving your answer in the form $a+b\sqrt{5}$ where a and b are rational numbers.

(3)

i) $\dfrac{\sqrt{180}-\sqrt{80}}{\sqrt{5}} = \dfrac{\sqrt{36\times5}-\sqrt{16\times5}}{\sqrt{5}} = \dfrac{\sqrt{36}\sqrt{5}-\sqrt{16}\sqrt{5}}{\sqrt{5}}$

$= \dfrac{6\sqrt{5}-4\sqrt{5}}{\sqrt{5}} = \dfrac{\cancel{\sqrt{5}}(6-4)}{\cancel{\sqrt{5}}} = 6-4 = 2$

$\therefore$ integer is 2.

ii) rationalising surds

$\dfrac{4\sqrt{5}-5}{7-3\sqrt{5}} \dfrac{\times(7+3\sqrt{5})}{\times(7+3\sqrt{5})} = \dfrac{(4\sqrt{5}-5)(7+3\sqrt{5})}{(7-3\sqrt{5})(7+3\sqrt{5})}$

$= \dfrac{28\sqrt{5}+(4\sqrt{5}\times3\sqrt{5})-35-15\sqrt{5}}{49+21\sqrt{5}-21\sqrt{5}+(-3\sqrt{5}\times3\sqrt{5})}$

$= \dfrac{13\sqrt{5}+(4\times3\times(5))-35}{49+(-3\times3\times5)} = \dfrac{13\sqrt{5}+60-35}{49-45} = \dfrac{13\sqrt{5}+25}{4}$

write in form $a+b\sqrt{5}$

$\therefore$ $\dfrac{25}{4}+\dfrac{13}{4}\sqrt{5}$ $\qquad a=\dfrac{25}{4} \qquad b=\dfrac{13}{4}$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

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Question 3 continued

(Total 5 marks)

Q3

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4.

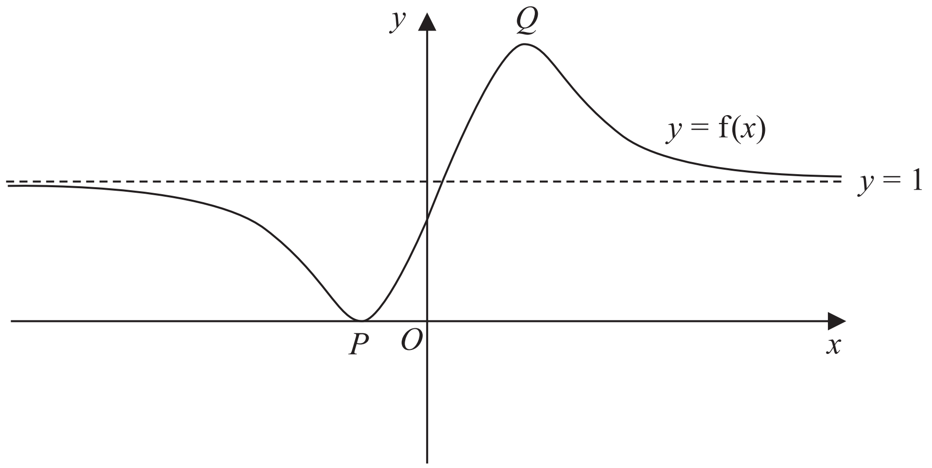


Figure 1

Figure 1 shows a sketch of a curve with equation $y = \mathrm{f}(x)$

The curve has a minimum at $P(-1, 0)$ and a maximum at $Q\left(\frac{3}{2}, 2\right)$

The line with equation $y = 1$ is the only asymptote to the curve.

On separate diagrams sketch the curves with equation

outside f(x) bracket ∴ only coordinates

(i) $y = \mathrm{f}(x) - 2$ → affected. Translation 2 units down $\binom{0}{-2}$ **(3)**

$P(-1,0) \rightarrow (-1, 0-2) = P(-1,-2)$

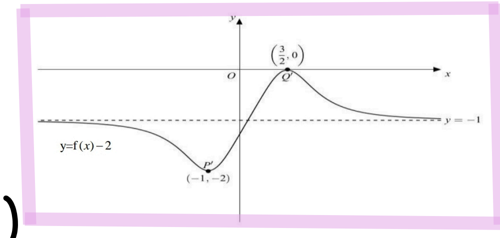
(ii) $y = \mathrm{f}(-x)$ $Q(\frac{3}{2}, 2) \rightarrow (\frac{3}{2}, 2-2) = Q(\frac{3}{2}, 0)$ **(3)**

Asymptote $y=1 \rightarrow y = 1-2 \Rightarrow y = -1$

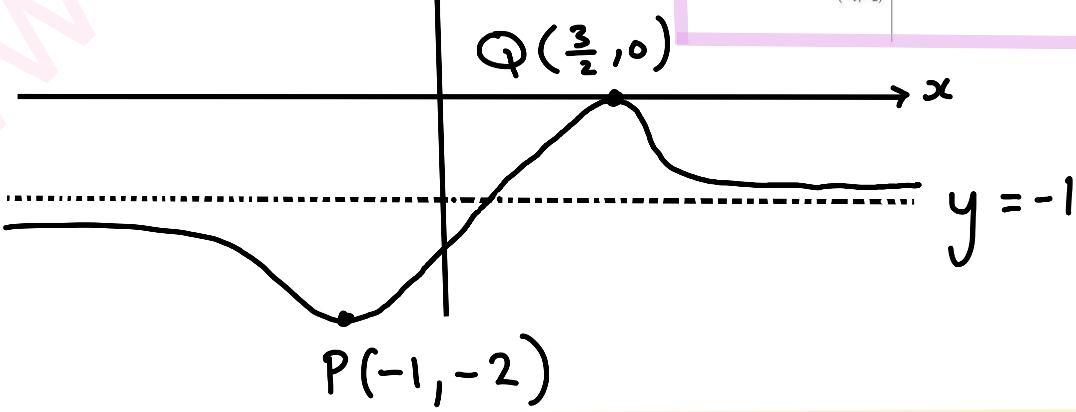
On each sketch you must clearly state

- the coordinates of the maximum and minimum points
- the equation of the asymptote

Mark scheme :



i) f(x) − 2



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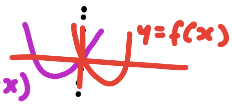
Question 4 continued

ii) $f(-x)$ is reflection of $f(x)$ in y-axis :

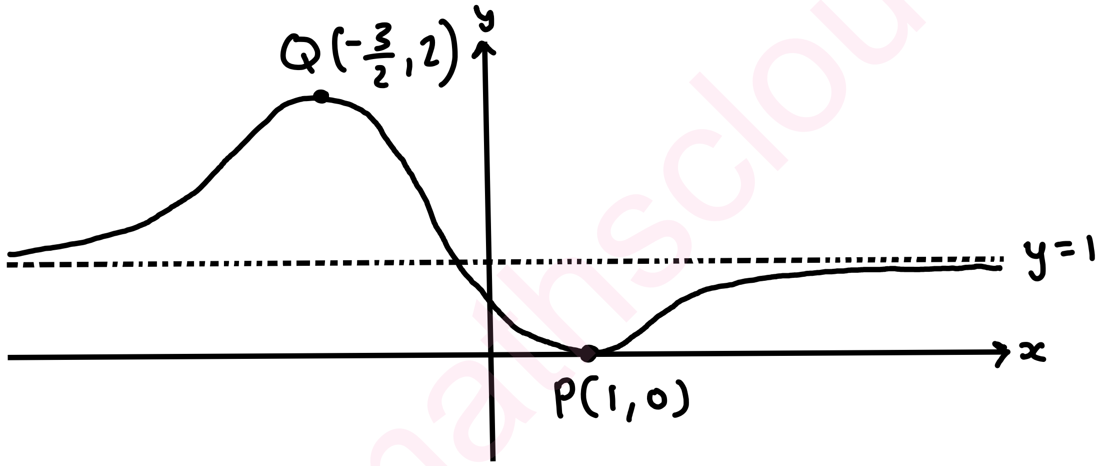


$\therefore \; P(-1,0) \rightarrow P(+1,0)$

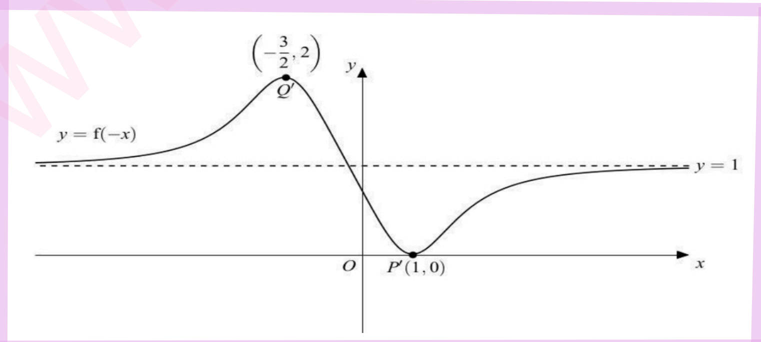
$Q\left(\frac{3}{2},2\right) \rightarrow Q\left(-\frac{3}{2},2\right)$

$y=1 \rightarrow y=1$

$f(-x)$



Mark scheme :



Q4

(Total 6 marks)

P 6 9 4 5 8 A 0 9 3 2

5. The curve C has equation $y = \mathrm{f}(x)$

Given that

- $\mathrm{f}(x)$ is a quadratic expression
- the maximum turning point on C has coordinates $(-2, 12)$
- C cuts the negative x-axis at -5

(a) find $\mathrm{f}(x)$

(4)

The line l_1 has equation $y = \dfrac{4}{5}x$

Given that the line l_2 is perpendicular to l_1 and passes through $(-5, 0)$

(b) find an equation for l_2, writing your answer in the form $y = mx + c$ where m and c are constants to be found.

(3)

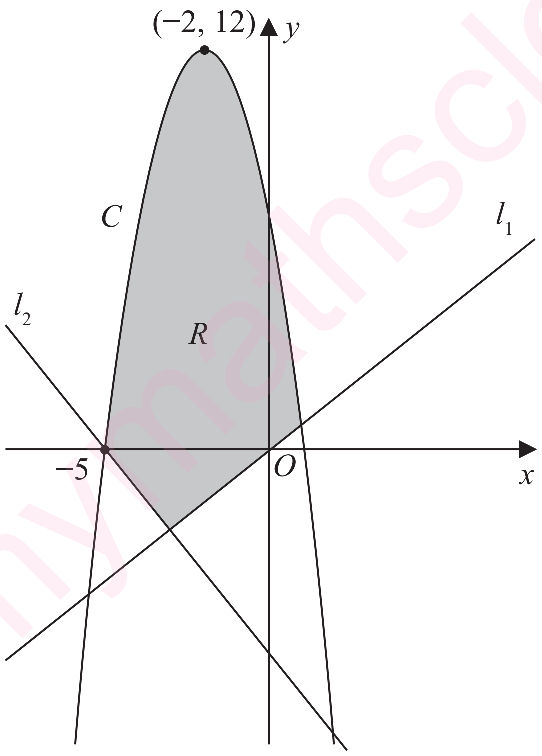


Figure 2

Figure 2 shows a sketch of the curve C and the lines l_1 and l_2

(c) Define the region R, shown shaded in Figure 2, using inequalities.

(2)

a) ① f(x) is quadratic $\therefore a(x+b)^2 + c$

② maximum turning point is is $(-2,12)$

Question 5 continued

to find coordinates of maximum point:

$y = (x+b)^2 + c$

inverse is x-coordinate $\quad$ y-coordinate

Explanation: when $y=c$: $c = (x+b)^2 + c$

$0 = (x+b)^2 \quad \therefore x = -b$

when $x=-b$: $y = (-b+b)^2 + c$

$\therefore y = c$

$\therefore a(x+2)^2 + 12$

③ Cuts negative x-axis at $x = -5$.

$\therefore$ when $x = -5$, $f(x) = 0$

$f(x) = a(x+2)^2 + 12$

$f(-5) = a(-5+2)^2 + 12 = 0$

$= a(-3)^2 + 12 = 0$

$= 9a + 12 = 0$

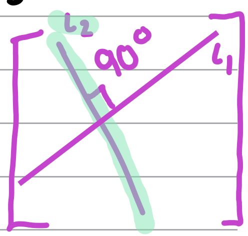
$-12 \quad 9a = -12 \quad -12$

$\div 9 \quad a = -\frac{12}{9} \quad \div 9$

$\therefore a = -\frac{4}{3}$

$\therefore f(x) = -\frac{4}{3}(x+2)^2 + 12$

$\therefore f(x) = 12 - \frac{4}{3}(x+2)^2 = -\frac{4}{3}x^2 - \frac{16}{3}x + \frac{20}{3}$



b) normal (l_2) is perpendicular to l_1

$\therefore$ we find gradient of normal (m_n) using perpendicular gradient rule $m_{normal} \times m_{l_1} = -1$

① gradient of l_1 : l_1: $y = \boxed{\frac{4}{5}}x + 0$

in $y = mx + c$ $\quad$ m is gradient $\quad \therefore m_{l_1} = \frac{4}{5}$

Question 5 continued

② find gradient of normal using formula.

$$M_n \times M_{l_1} = -1$$

$$\div \tfrac{4}{5} \left(M_n \times \frac{4}{5} = -1 \right) \div \frac{4}{5}$$

$$M_n = -\frac{5}{4}$$

③ find equation of l_2 using line passing through (a, b) & gradient m

equation : $(y - b) = m(x - a)$

$a = -5$
$b = 0$
$m = -\frac{5}{4}$

$$(y - 0) = -\frac{5}{4}(x - (-5))$$

④ write in the form $y = mx + c$

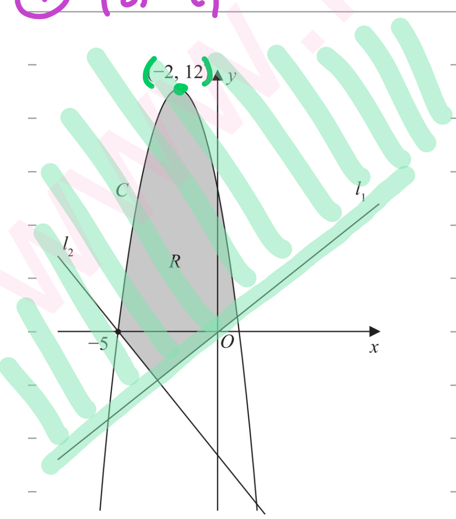
$$y - 0 = -5/4\,(x + 5)$$

$$\therefore\ y = -\frac{5}{4}x - \frac{25}{4}$$

c) Three inequalities for l_1 & l_2 & C

① for l_1 :

$l_1 :\ y = \frac{4}{5}x$



To find inequality , Use point INSIDE Valid region & Substitute into equation l_1 & make it TRUE.

Use point $(-2, 12)$

$$y = \frac{4}{5}x$$

$$12 = \frac{4}{5}(-2)$$

$$12 = -\frac{8}{5}$$

To make it TRUE : $12 \geq 1.6$

$$\therefore\ y \geq \frac{4}{5}x$$

$\geq$ Not $>$ $\because$ lines are solid & not dashed *

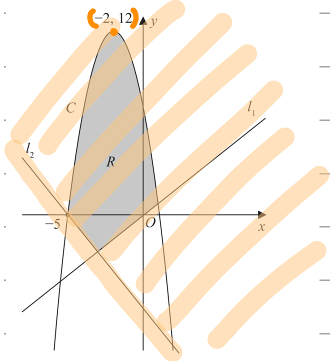
solid dashed

P 6 9 4 5 8 A 0 1 2 3 2

Question 5 continued

← Part (b)

② for l_2: $\quad l_2 : y = -\frac{5}{4}x - \frac{25}{4}$



To find inequality, Use point INSIDE Valid region & Substitute into equation l_1 & make it TRUE.

Use point $(-2, 12)$

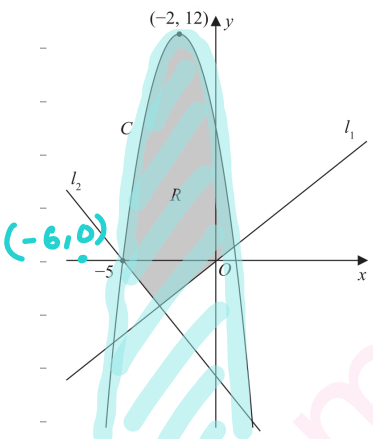
$$y = -\frac{5}{4}x - \frac{25}{4}$$
$$12 = -\frac{5}{4}(-2) - \frac{25}{4}$$
$$12 = -3.75$$

To make it TRUE: $12 \geqslant -3.75$

$$\therefore\ y \geqslant -\frac{5}{4}x - \frac{25}{4}$$

from part (a)

③ Curve C: $\quad C: y = 12 - \frac{4}{3}(x+2)^2$



To find inequality, Use point OUTSIDE Valid region & Substitute into equation l_1 & make it FALSE.

Use point $(-6, 0)$

$$y = 12 - \frac{4}{3}(x+2)^2$$
$$0 = 12 - \frac{4}{3}(-6+2)^2$$
$$0 = -\frac{28}{3}$$

To make it FALSE: $0 \leqslant -\frac{28}{3}$

$$\therefore\ y \leqslant 12 - \frac{4}{3}(x+2)^2$$
$$y \leqslant -\frac{4}{3}x^2 - \frac{16}{3}x + \frac{20}{3}$$

if point outside valid region is used: make inequality FALSE
if point inside valid region is used: make inequality TRUE

$\therefore$ region R is defined by inequalities:

$$y \geqslant \frac{4}{5}x \qquad y \geqslant -\frac{5}{4}x - \frac{25}{4} \qquad y \leqslant -\frac{4}{3}x^2 - \frac{16}{3}x + \frac{20}{3}$$

Q5

(Total 9 marks)

Leave blank

Turn over

P 6 9 4 5 8 A 0 1 3 3 2

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Given that

$$2xy - 3x^2 = 50$$

and

$$y - x^3 + 6x = 0$$

show that

$$2x^4 - 15x^2 - 50 = 0$$

(2)

(b) Hence solve the simultaneous equations

$$2xy - 3x^2 = 50$$
$$y - x^3 + 6x = 0$$

Give your answers in fully simplified surd form.

(5)

a) ① rearrange both equations given to make y the subject

1) $+3x^2$ $\quad 2xy - 3x^2 = 50 \quad$ $+3x^2$

$2xy = 50 + 3x^2$

$\div 2x$ $\quad y = \dfrac{50+3x^2}{2x} \quad$ $\div 2x$

2) $\quad y - x^3 + 6x = 0$

$+x^3$ $\quad y + 6x = x^3 \quad$ $+x^3$

$-6x$ $\quad y = x^3 - 6x \quad$ $-6x$

② equate the two equations together

$\dfrac{50+3x^2}{2x} = x^3 - 6x$

$\times 2x$ $\quad 50 + 3x^2 = 2x^4 - 12x^2 \quad$ $\times 2x$

$-3x^2$ $\quad 50 = 2x^4 - 15x^2 \quad$ $-3x^2$

-50 $\quad 0 = 2x^4 - 15x^2 - 50 \quad$ -50

$\therefore 2x^4 - 15x^2 - 50 = 0$

Question 6 continued

b) Can solve by letting $x^2 = b$

$x^4 = x^{2\times 2} = (x^2)^2 = b^2$

$\therefore$ let $2x^4 - 15x^2 - 50 = 0$ become:

$2b^2 - 15b - 50 = 0$

Factorise: $(2b+5)(b-10) = 0$

Solve: $2b+5=0$ $\quad$ $b-10=0$

$2b = -5$ $\quad$ $b = 10$

$b = -5/2$

$b = x^2 \neq -5/2$

UNDEFINED $\because$ Cannot have square number that is negative

$\underbrace{b = x^2 = 10}$

square root $\quad x^2 = 10$

$x = \pm\sqrt{10}$ $\quad$ square root

Find y values:

$y = x^3 - 6x$

when $x = +\sqrt{10}$ $\quad y = (+\sqrt{10})^3 - 6(+\sqrt{10})$

$= 10\sqrt{10} - 6\sqrt{10} = 4\sqrt{10}$

when $x = -\sqrt{10}$ $\quad y = (-\sqrt{10})^3 - 6(-\sqrt{10})$

$= -10\sqrt{10} + 6\sqrt{10} = -4\sqrt{10}$

$\therefore$ $x_1 = \sqrt{10}$, $y_1 = 4\sqrt{10}$ & $x_2 = -\sqrt{10}$, $y_2 = -4\sqrt{10}$

P 6 9 4 5 8 A 0 1 5 3 2

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 7 marks)

7. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$

Given that

- $\mathrm{f}'(x) = \dfrac{2}{\sqrt{x}} + \dfrac{A}{x^2} + 3$, where A is a constant
- $\mathrm{f}''(x) = 0$ when $x = 4$

(a) find the value of A.

(4)

Given also that

- $\mathrm{f}(x) = 8\sqrt{3}$, when $x = 12$

(b) find $\mathrm{f}(x)$, giving each term in simplest form.

(5)

$$f(x) \underset{\text{integrate}}{\overset{\text{differentiate}}{\rightleftarrows}} f'(x) \underset{\text{integrate}}{\overset{\text{differentiate}}{\rightleftarrows}} f''(x)$$

a) $f'(x) \longrightarrow f''(x)$

1 write $f'(x)$ in easier form for differentiation.

$$f'(x) = \frac{2}{\sqrt{x}} + \frac{A}{x^2} + 3 = 2\,\frac{1}{\sqrt{x}} + A\,\frac{1}{x^2} + 3$$

$$= 2\,\frac{1}{x^{\frac{1}{2}}} + A\,\frac{1}{x^2} + 3$$

1 indicies rule : $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

$\because x^0 = 1$

$$= 2\,\frac{1}{x^{\frac{1}{2}}} + A\,\frac{1}{x^2} + 3 \qquad = 2x^{-\frac{1}{2}} + Ax^{-2} + 3x^0$$

2 indicies rule : $\dfrac{a}{x^b} = ax^{-b}$

$$\therefore f'(x) = 2x^{-\frac{1}{2}} + Ax^{-2} + 3$$

2 differentiate

$$f''(x) = (-\tfrac{1}{2})(2x^{-1/2-1}) + (-2)(Ax^{-2-1}) + 0\,(3x^{0-1})$$

$$= -x^{-3/2} - 2Ax^{-3}$$

3 When $x = 4$, $f''(x) = 0$. Substitute into equation.

$$f''(4) = -(4)^{-3/2} - 2A(4)^{-3} = 0$$

Question 7 continued

4 Solve for A.

$$-(4)^{-3/2} - 2A(4)^{-3} = 0$$

$$-\frac{1}{8} - \frac{2A}{64} = 0$$

$+\frac{1}{8}$ $+\frac{1}{8}$

$$\frac{-2A}{64} = \frac{1}{8}$$

$\times 64$ $\times 64$

$$-2A = 8$$

$\div -2$ $\div -2$

$$A = -4$$

$$\therefore A = -4$$

b) $f'(x) \longrightarrow f(x)$

1 write $f'(x)$ in easier form for integration

$$f'(x) = \frac{2}{\sqrt{x}} - \frac{4}{x^2} + 3$$

from part (a): $A = -4$

$$f'(x) = 2x^{-1/2} - 4x^{-2} + 3$$

already did this in part (a).

2 Integration.

$$f(x) = \int f'(x)\,dx = \int 2x^{-1/2} - 4x^{-2} + 3x^0\,dx$$

$$= \left[\left(\frac{2}{-\frac{1}{2}+1}x^{-1/2+1}\right) + \left(\frac{-4}{-2+1}x^{-2+1}\right) + \left(\frac{3}{0+1}x^{0+1}\right)\right]$$

$$= 4x^{\frac{1}{2}} + 4x^{-1} + 3x + c$$

3 When $x = 12$, $f(x) = 8\sqrt{3}$. Substitute this into equation.

$$f(12) = 4(12)^{1/2} + 4(12)^{-1} + 3(12) + c = 8\sqrt{3}$$

4 find value of C.

$$4(12)^{1/2} + 4(12)^{-1} + 3(12) + c = 8\sqrt{3}$$

$$4\sqrt{12} + \frac{4}{12} + 36 + c = 8\sqrt{3}$$

$$4\sqrt{4\times 3} + \frac{4(1)}{4(3)} + 36 + c = 8\sqrt{3}$$

P 6 9 4 5 8 A 0 1 9 3 2

Question 7 continued

$$4(2)\sqrt{3} + \frac{1}{3} + 36 + c = 8\sqrt{3}$$

$$8\sqrt{3} + \underbrace{\frac{1}{3} + 36} + c = 8\sqrt{3}$$

$$8\sqrt{3} + \frac{109}{3} + c = 8\sqrt{3}$$

$-8\sqrt{3}$ $\quad$ $-8\sqrt{3}$

$$\frac{109}{3} + c = 0$$

$-\frac{109}{3}$ $\quad$ $-\frac{109}{3}$

$$c = -\frac{109}{3}$$

5 Write $f(x)$

$$\therefore f(x) = 4x^{1/2} + 4x^{-1} + 3x - \frac{109}{3}$$

$4x^{-1}$ can be written as $\frac{4}{x}$

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Question 7 continued

Q7

(Total 9 marks)

8.

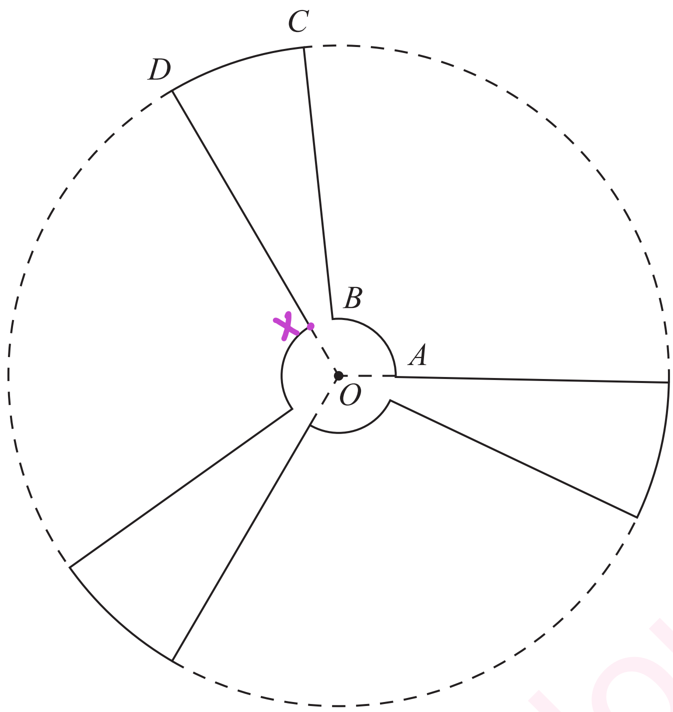


Figure 3

Figure 3 shows a sketch of the outline of the face of a ceiling fan viewed from below.

The fan consists of three identical sections congruent to $OABCDO$, shown in Figure 3, where

- $OABO$ is a sector of a circle with centre O and radius 9 cm
- $OBCDO$ is a sector of a circle with centre O and radius 84 cm
- angle $AOD = \dfrac{2\pi}{3}$ radians units!!

Given that the length of the arc AB is 15 cm,

(a) show that the length of the arc CD is 35.9 cm to one decimal place.
(3)

The face of the fan is modelled to be a flat surface.

Find, according to the model,

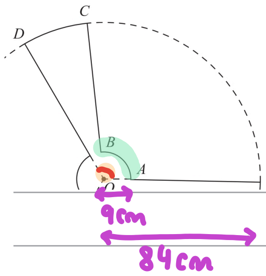
(b) the perimeter of the face of the fan, giving your answer to the nearest cm,
(2)

(c) the surface area of the face of the fan.

Give your answer to 3 significant figures and make your units clear.
(5)

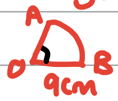
Question 8 continued

a) length of arc : $S = r\theta$



Arc AB is 15cm & angle AOD is $\frac{2\pi}{3}$ rad.
Use this to find $\angle DOC$ for arc CD length

$S = r\theta$



$15 = 9 \times \angle AOB$ $\quad \div 9$

$\frac{15}{9} = \angle AOB$

$\frac{5}{3} = \angle AOB$

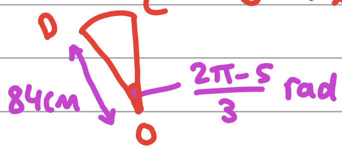
$\angle AOD = \angle AOB + \angle DOC$

$2\pi/3 = 5/3 + \angle DOC$ $\quad -5/3$

$2\pi/3 - 5/3 = \angle DOC$

$\therefore \angle DOC = \frac{2\pi - 5}{3}$ rad

$S = r\theta$



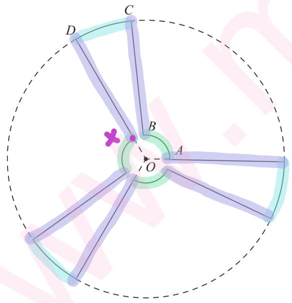
$S = 84 \times \frac{2\pi - 5}{3} = 56\pi - 140$

$= 35.92918\ldots \approx 35.9$

$\therefore$ length arc CD = 35.9 cm (1dp)

b) Perimeter of fan = $3 \times (XD + DC + CB + BA)$

$\because$ fan is split into $\underline{\underline{3}}$ identical OABCDO



length 'XD' = CB.

OB = 9cm $\quad$ 9

OC = 48cm $\quad$ 84

$\therefore BC = OC - OB = 84 - 9 = 75$.

from part (a)

$\therefore$ Perimeter $= 3 \times (75 + 35.9 + 75 + 15) = 3(200.9) = 602.7$

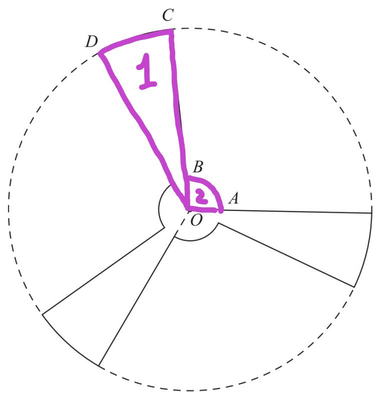
$\therefore$ Perimeter = 603 cm (nearest cm)

Turn over

P 6 9 4 5 8 A 0 2 3 3 2

Question 8 continued

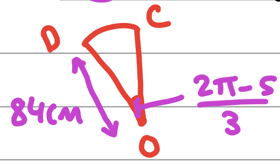
c) For surface area, can split OABCDO into 2 shapes.
Sector CDO & Sector AOB.



$\therefore$ Area $= 3 \times$ (Area CDO + Area AOB)
$\because$ fan is split into $\underline{\underline{3}}$ identical OABCDO

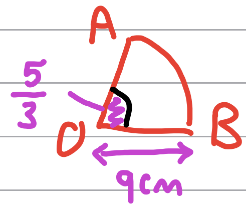
Sector Area : $A = \frac{1}{2} r^2 \theta$

① Area CDO $= \frac{1}{2} \times 84^2 \times \frac{2\pi - 5}{3}$ → from Part (a)



$= 2352\pi - 5880$
$= 1509.025921$
$\approx 1509 \text{ cm}^2$

② Area AOB $= \frac{1}{2} \times 9^2 \times \frac{5}{3}$
$= \frac{135}{2} = 67.5 \text{ cm}^2$



$\therefore$ Total Area $= 3(1509 + 67.5) = 3(1576.5) = 4729.5$

$\therefore$ Surface Area of face of fan $= 4730 \text{ cm}^2$ (3 sig fig)
or 0.473 m^2

P 6 9 4 5 8 A 0 2 4 3 2

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Question 8 continued

Q8

(Total 10 marks)

Leave blank

9.

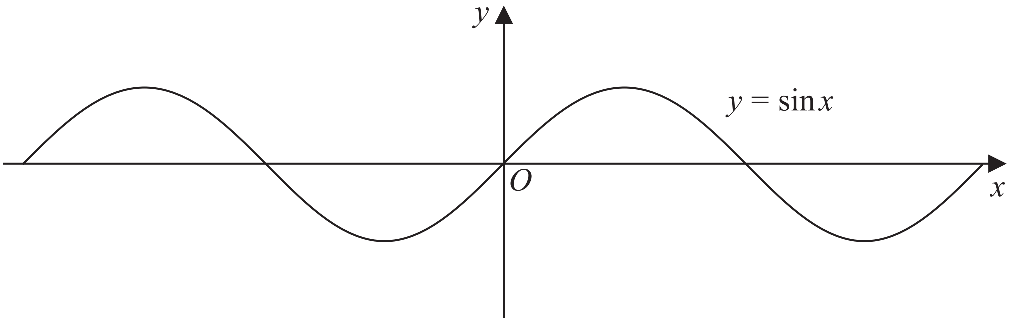


Figure 4

Figure 4 shows part of the graph of the curve with equation $y = \sin x$

Given that $\sin\alpha = p$, where $0 < \alpha < 90°$ units

(a) state, in terms of p, the value of

(i) $2\sin(180° - \alpha)$

(ii) $\sin(\alpha - 180°)$

(iii) $3 + \sin(180° + \alpha)$

(3)

A copy of Figure 4, labelled Diagram 1, is shown on page 27.

On Diagram 1,

(b) sketch the graph of $y = \sin 2x$

(2)

(c) Hence find, in terms of α, the x coordinates of any points in the interval $0 < x < 180°$ where

$$\sin 2x = p$$

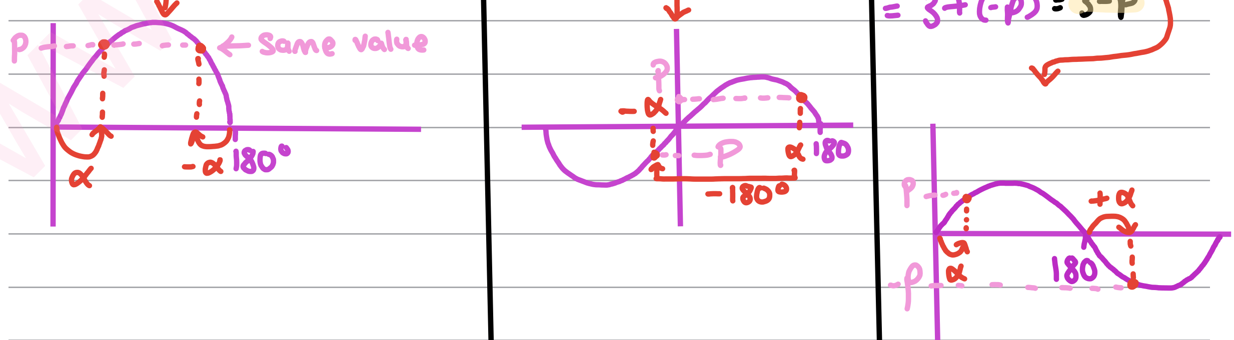
(3)

a)

i) $2\sin(180-\alpha) = 2\times p = 2p$

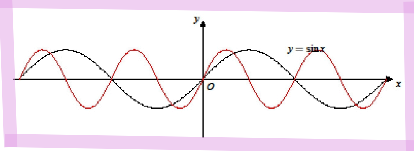
ii) $\sin(\alpha-180) = -p$

iii) $3+\sin(180+\alpha) = 3+(-p) = 3-p$



P 6 9 4 5 8 A 0 2 6 3 2

Mark Scheme :



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Question 9 continued

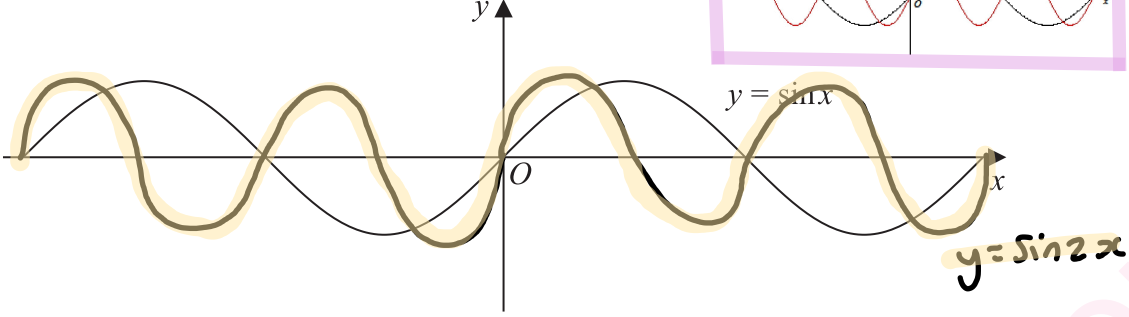


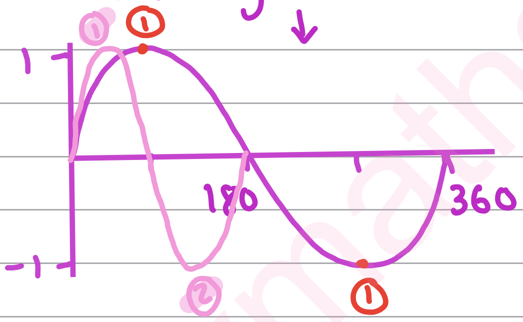
Diagram 1

b) Sketching $y = \sin 2x$. if $\sin x = f(x)$, $\sin 2x = f(2x)$

$f(2x)$ is horizontal squash by factor 2 (multiply x-coordinates by $\frac{1}{2}$)

y-coordinates unchanged $\because$ inside $f(x)$ brackets only affects x-coordinates

$\therefore$ Same height graph, BUT every $180°$ has 2 turning points instead of 1 like in $y = \sin x$



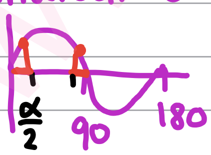
c) $\sin \alpha = p$ so when $\sin 2x = p$

$2x = \alpha$

$\therefore x = \frac{\alpha}{2}$

in interval $0 < x < 180°$, 2 solutions for $\sin 2x = p$.

$\therefore$



first solution is $x = \alpha/2$

$\therefore$ second solution will be $x = 90 - \frac{\alpha}{2}$

$\therefore x = \frac{\alpha}{2}$ & $x = 90° - \frac{\alpha}{2}$

Q9

(Total 8 marks)

10.

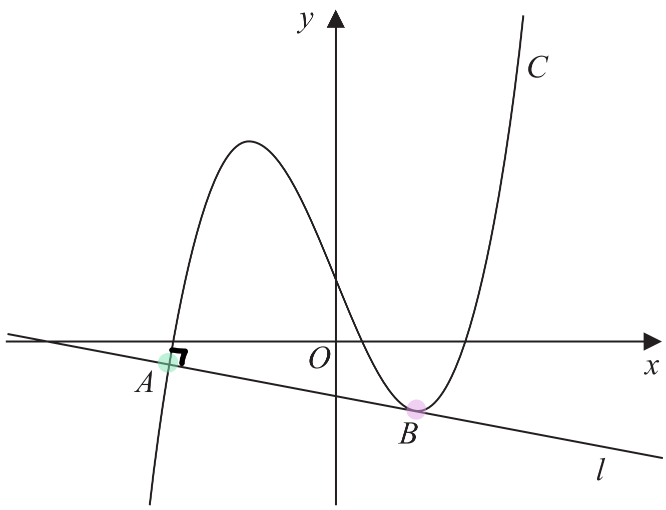


Figure 5

Figure 5 shows a sketch of the curve C with equation

$$y = \frac{2}{7}x^3 + \frac{1}{7}x^2 - \frac{5}{2}x^1 + kx^0 \quad \because x^0 = 1$$

where k is a constant.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(2)

The line l, shown in Figure 5, is the normal to C at the point A with x coordinate $-\frac{7}{2}$

Given that l is also a tangent to C at the point B,

(b) show that the x coordinate of the point B is a solution of the equation

$$12x^2 + 4x - 33 = 0$$

(4)

(c) Hence find the x coordinate of B, justifying your answer.

(2)

Given that the y intercept of l is -1

(d) find the value of k.

(4)

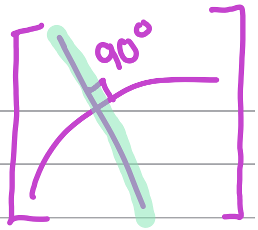
a) $dy/dx = 3\left(\frac{2}{7}x^{3-1}\right) + 2\left(\frac{1}{7}x^{2-1}\right) + 1\left(-\frac{5}{2}x^{1-1}\right) + 0\left(kx^{0-1}\right)$

$= \frac{6}{7}x^2 + \frac{2}{7}x - \frac{5}{2}$

P 6 9 4 5 8 A 0 2 8 3 2

Question 10 continued

b) normal is Perpendicular to Curve



∴ we find gradient of normal (m_n)
using Perpendicular gradient rule $m_{normal} \times m_{curve} = -1$

① find gradient of curve C by putting $x = -\frac{7}{2}$ into gradient function dy/dx

$$\left.\frac{dy}{dx}\right|_{x=-\frac{7}{2}} = \frac{6}{7}\left(-\frac{7}{2}\right)^2 + \frac{2}{7}\left(-\frac{7}{2}\right) - \frac{5}{2} = 7$$

② find gradient of line l.

$$m_n \times m_c = -1$$
$$m_n \times 7 = -1$$
$$\therefore m_n = -1/7$$

③ tangent means gradient of tangent is same as gradient of equation

As line l is tangent to curve C at B, at x-value of B, $dy/dx = -1/7$

$$\left.\frac{dy}{dx}\right|_{x=?} = \frac{6}{7}x^2 + \frac{2}{7}x - \frac{5}{2} = -\frac{1}{7}$$

$\times 7$ $6x^2 + 2x - 35/2 = -1$ $\times 7$

$\times 2$ $12x^2 + 4x - 35 = -2$ $\times 2$

$+2$ $12x^2 + 4x - 33 = 0$ $+2$

∴ x-coordinate of B is a solution of the equation:
$$12x^2 + 4x - 33 = 0$$

c) to find x-coordinate of B, solve $12x^2 + 4x - 33 = 0$ from part (b)

$$12x^2 + 4x - 33 = 0$$

Factorise: $(2x - 3)(6x + 11) = 0$

Solve: $2x - 3 = 0$ $6x + 11 = 0$

$2x = 3 \Rightarrow \therefore x = \frac{3}{2}$ $6x = -11 \Rightarrow \therefore x = \frac{-11}{6}$

Question 10 continued

B in figure 5 is in 4^{th} quadrat where

$$\begin{array}{c|c} ② & ① \\ \hline ③ & ④ \end{array}$$

x-coordinates are positive.

$\therefore$ x-coordinate of B : $x = \frac{3}{2}$

d) y intercept of l is -1. Gradient (m_n) of l is $-1/7$. from part (b)

$\therefore$ using equation of line is $y = mx + c$

equation of l: $y = -1/7\, x - 1$

At point A, $x = -7/2$, both C and l intersect.

find y-coordinate of A using the equation of l we found:

$$y = -1/7\,(-7/2) - 1 = 1/2 - 1 = -1/2$$

Substitute y into equation of C.

$$-1/2 = \frac{2}{7}x^3 + \frac{1}{7}x^2 - 5/2\, x + k$$

Substitute x into equation of C.

$$-1/2 = \frac{2}{7}(-7/2)^3 + 1/7\,(-7/2)^2 - 5/2\,(-7/2) + k$$

$$-1/2 = -49/4 + 7/4 + 35/4 + k$$

$$-1/2 = -7/4 + k$$

Solve for k:

$$-1/2 = -7/4 + k$$

$+7/4$ $\quad 5/4 = k \quad$ $+7/4$

$$\therefore k = \frac{5}{4}$$

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Question 10 continued

Question 10 continued

Q10

(Total 12 marks)

TOTAL FOR PAPER IS 75 MARKS

END